giving his valuable time in generating some of the results and the computer plots for this paper.

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Effects of Axial and Radial Air Injection on the Near Wake with and without External Compression

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Introduction

EXPERIMENTS and analysis show that the base pressure of blunt-based bodies at supersonic speeds can be increased by initiating a zone of combustion adjacent to the base region. Efficient operation has been demonstrated using the concept of "base burning" in which fuel reacts in the near-wake shear flow.[1,2] Experience shows, however, that the maximum base pressure with this mode of operation approximately equals the freestream pressure and that efficiency decreases with increasing base pressure. Base thrust (i.e., base pressure higher than the freestream pressure) is possible using "external burning" in which the fuel combusts in the exterior inviscid flow surrounding the wake.[3,4] The economics of this mode of operation, however, remains uncertain. It is currently thought that a combination of base and external burning offers the greatest promise for achieving base thrust. Although evidence indicates that these modes of burning can be superimposed,[5] efficient performance is yet to be verified.

Results from the second phase of an experimental program to study base pressure enhancement are reported in the present Note. Reference 6 documents the first phase of testing which defines base pressure trends and near-wake structure detail for systematic variations in the freestream flowfield simulating external burning. Freestream manipulation was achieved with a series of axisymmetric and asymmetric test section wall contours translated relative to the model base plane. The contours were designed to focus compression fields on the near wake, representing some reasonable total heat addition in the exterior flow at different rates of release. Further base pressure and near-wake structure measurements are presented here both for radial and base injection of cold air. These tests were conducted to evaluate flow phenomena arising from actual injection of a secondary fluid and are intended to link the first phase simulation tests with the final program segment in which external and base burning of hydrogen fuel will be investigated.

Test Facility

The axisymmetric test facility was designed to simulate the base flow of a projectile at Mach 3 with a fineness ratio of about 6 and a Reynolds number of 3×10^6. The 5.72-cm-diam model is supported in the upstream subsonic flow and extends through the throat into the test section. Bleed air is ducted into the model through the upstream support struts. Radial jet injection was done though six sonic nozzles located about 0.25 base radii upstream of the base plane and equally spaced around the periphery. A porous, sintered-metal base plate was used for axial injection into the near wake. Details of the test facility, other instrumentation, and experimental techniques and accuracies are discussed in Ref. 6.

Results and Discussion

Radial Jet Injection

The effect of radial, cold-air injection on the base pressure is shown in Fig. 1. Results are shown both for no external compression and for axisymmetric external compression with Compression Section II[6] (which produces a net thrust with no cold-air injection). The important observation is that base pressure P_b decreases with injection rate as a result of the competing effects of vortex generation, flow displacement, and degradation in total pressure by the attendant shock system. The decrease is essentially independent of the nozzle diameter and reaches a maximum of over 10% of the freestream pressure P_l. At lower injection rates the loss in base pressure is somewhat reduced by compression.

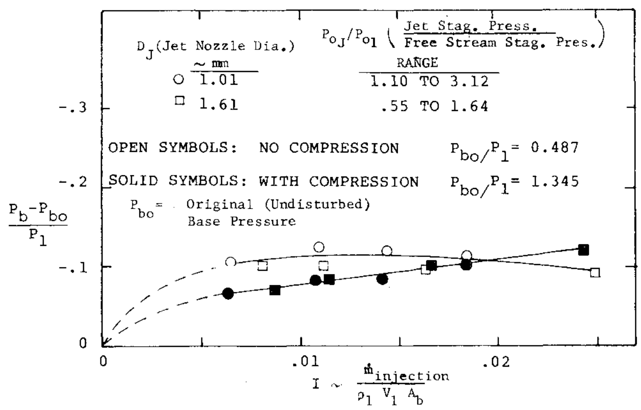

Fig. 1 Effect of radial jet injection on base pressure.

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Index categories: Jets, Wakes, and Viscid-Inviscid Flow Interactions; Airbreathing Propulsion.

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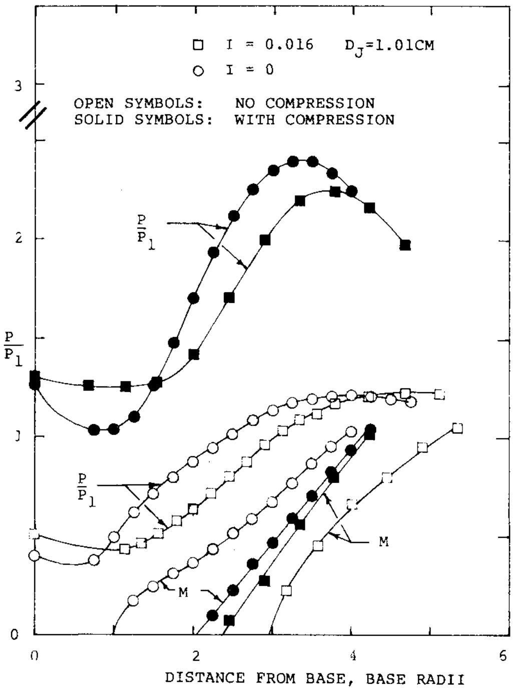

Fig. 2 Near-wake centerline static pressure and Mach number distributions with and without radial injection.

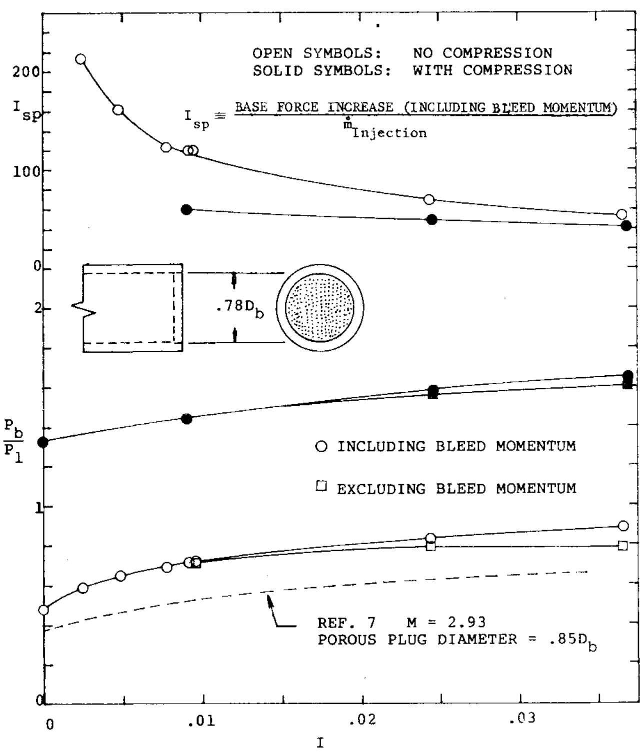

Fig. 3 Base pressure and specific impulse for base injection.

Typical pressure and Mach number distributions along the centerline of the near wake with and without radial injection are compared in Fig. 2. Injection significantly alters the wake structure and, for the case of no compression, moves the rear stagnation and sonic points considerably closer to the base. With compression, the stagnation and sonic point locations are apparently controlled by the length scales of the external compression zone. Radial surveys between and behind jets at four axial locations in the near wake (upstream and downstream of the rear stagnation point) reveal that the jets develop into wakes embedded in the outer layers of the base-wake shear flow. It is not clear why radial injection significantly reduces the base pressure and alters the wake structure, but it is possible that vortices created by boundary-layer separation and jet rollup due to blockage intensify the turbulent mixing in the near wake. Similar trends with slightly less magnitude were reported in Ref. 6 using radial pegs to model the jet blockage effects. It is evident that the effects of discrete jet injection must be incorporated in a realistic analytical model of the near wake for a real external combustion system.

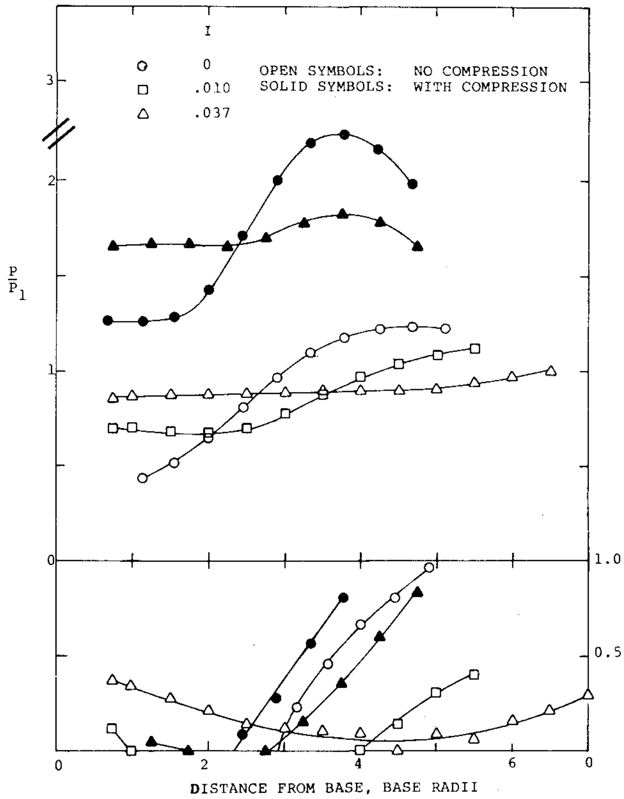

Fig. 4 Near-wake centerline static pressure and Mach number distributions with and without base injection.

Axial Base Injection

Figure 3 presents results for base injection with and without external compression. The lower portion of the figure shows the effective base pressures including and excluding the momentum flow rate of the bleed air. For comparison with the no compression results, the figure also includes the Mach 2.93 wind tunnel data of Freeman and Korkegi[7] for an ogive-cylinder projectile with air injection through a porous base. The elevation in base pressure due to base injection is comparable for these two sets of data. The actual base-to-freestream pressure ratios differ because of the nose effect in the results of Ref. 7 and the slight wake compression induced by boundary-layer growth on the outer ducting for the present tests.

The effectiveness of cold-air base injection to elevate the base pressure is indicated by the specific impulse results in the upper portion of Fig. 3. As is well established, relatively high values of specific impulse are obtainable at low injection rates. The effectiveness of injection decreases rapidly with injection rate, however. As previously mentioned, external burning (external compression) or the combination of external burning and base burning offer the potential for producing base thrust. These cold-flow results show that the effectiveness of base injection is diminished as a result of external compression and, thus, suggest that the effects of external burning and base burning are not simply additive.

Pressure and Mach number distributions along the centerline of the near wake are shown in Fig. 4. For the case of no external compression, the length of the near wake increases markedly with injection rate. For $I=0.01$ the recirculation bubble extends from about 1 to 4 base radii downstream of the base. Radial surveys slightly upstream of this recirculation bubble reveal a uniform bleed flow extending over 90% of the base diameter. For $I=0.037$ the recirculation bubble is blown off, and the axial pressure gradients along the wake are extremely small. Beyond $I=0.037$ only a small increase in the near-wake static pressure can be expected, since it must approach P_l as I approaches 1.

With external compression, the near-wake length is predominately controlled by the length scale of the compression region. Again, for $I=0.037$ the axial pressure gradients are small and apparently the wake is nearly fully opened by the bleed flow. Although a weak recirculation bubble is indicated by the data, this is not within the demonstrated $\pm 1\%$ data accuracy. The rapid acceleration the centerline velocity to sonic speed is aided by the externally generated expansion simulating a tailoff of combustion as well as increased turbulent mixing.

Acknowledgment

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A Modified One-Equation Model of Turbulence for the Calculation of Free Shear Flows

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Nomenclature

C_D = coefficient of dissipation of turbulent kinetic energy
k = turbulent kinetic energy
k_m = maximum value of k at certain x
h_s = empirical constant, $h_s = \Delta\delta$
ℓ = length-scale of turbulence
u = local mean velocity in x direction
U = freestream velocity
x,y = distances measured in the mainstream direction and the direction normal to it respectively
x_i,x_f = initial and final values of x for flow region considered
ρ = density
$\nu_{\rm eff}$ = effective viscosity
δ = boundary-layer thickness defined at $u = U + 0.01\epsilon$
$\delta_{1/2}$ = value of y where $u = U + 0.5\epsilon$
τ = local shear stress (turbulent)
λ = nondimensional length-scale of turbulence, $=\ell/\delta$
σ_k = Schmidt number of turbulent kinetic energy
Δ = boundary-layer thickness for the linear approximation of the velocity profile
ϵ = maximum velocity difference across the layer

Subscripts

i = condition at x_i

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Introduction

THE use of a turbulence model to evaluate the effective viscosity is common in the calculation of free shear flows. Models generally describe this quantity as proportional to the product of the length-scale and a velocity-scale of turbulence

$$\nu_{\rm eff} = \ell k^{1/2} \tag{1}$$

The molecular viscosity is neglected in Eq. (1), and the length-scale is defined such that the proportionality constant is unity. Values of ℓ and k are to be determined either empirically or by solving certain conservation equations. Models that require the solution of zero, one, or two equations are well known and have been used for the calculation of different turbulent flows.[1-5] As the number of equations is increased, the generality of a turbulence model is generally improved on at the expense of its simplicity.

The one-equation model employs an empirical variation for the length-scale of turbulence. The present work is to improve the generality of this model by the derivation of a semiempirical expression for the length-scale using an integral analysis, with some approximations, applied to two-dimensional free shear flows.

Analysis

The length-scale of turbulence can be expressed as:

$$\ell = \lambda\delta \tag{2}$$

where λ is normally taken as an empirical constant. This expression can be improved if λ is related to the mean flow characteristics; to obtain such relation, the integral forms of the governing equations are analytically solved. In order to simplify the algebraic solution, approximate shapes are assumed for different profiles to evaluate the integral quantities. A linear mean velocity profile is assumed, while the turbulent kinetic energy profile is taken uniform across the layer. If we consider the general shape of the velocity profile for a flow remote from walls, it could be linearly approximated as shown in Fig. 1. We may consider the relation

$$\Delta = h_s\delta \tag{3}$$

where h_s is a quantity of the order of unity whose value is to be determined later.